Multivariable Calculus

Quiz 3 **SOLUTIONS**

1) For the following augmented matrices, determine which are in RREF.

a) $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$ b) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{array}\right]$ c) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 15 \end{array}\right]$

Solution: The matrices in a) and c) are in RREF. The matrix in b) fails to be in RREF because the leading one in the far right entry of the bottom row is not the only non-zero entry in its column.

2) The following matrices are in RREF. If there is a unique solution, state what it is. If there are an infinite number of solutions, state a free variable and give an equation for the other variables in terms of the free one. Otherwise, say there is no solution. You may assume the variables are x, y, and z.

a) $\left[\begin{array}{ccc|c} 1 & 0 & -4 & 2 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Solution: This system has an infinite number of solutions. We can take z to be the free variable. The first two rows give

$$x = 2 + 4z,$$
$$y = 5 - z.$$

b) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 0 \end{array}\right]$

Solution: The system has a unique solution: $x = 1, y = -4$, and $z = 0$.

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3) Solve the following system by writing it as an augmented matrix and row reducing to RREF.

$$\begin{aligned} 2x + 3y &= 1 \\ -x + 5y &= 6 \end{aligned}$$

Solution:

$$\left[\begin{array}{cc|c} 2 & 3 & 1 \\ -1 & 5 & 6 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -5 & -6 \\ 2 & 3 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -5 & -6 \\ 0 & 13 & 13 \end{array}\right]$$
$$\sim \left[\begin{array}{cc|c} 1 & -5 & -6 \\ 0 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 1 \end{array}\right]$$

So the unique solution is $x = -1$ and $y = 1$.

4) Solve the following system by writing it as an augmented matrix and row reducing to RREF.

$$\begin{aligned} 2y + 3z &= 8 \\ 2x + 3y + z &= 5 \\ x - y - 2z &= -5 \end{aligned}$$

Solution:

$$\left[\begin{array}{ccc|c} 0 & 2 & 3 & 8 \\ 2 & 3 & 1 & 5 \\ 1 & -1 & -2 & -5 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & -2 & -5 \\ 0 & 2 & 3 & 8 \\ 2 & 3 & 1 & 5 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & -2 & -5 \\ 0 & 2 & 3 & 8 \\ 0 & 5 & 5 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & -2 & -5 \\ 0 & 1 & 1 & 3 \\ 0 & 2 & 3 & 8 \end{array}\right]$$
$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

So, the unique solution is $x = 0, y = 1$, and $z = 2$.